



Christ Church  
Grammar School

2024

TEST 3

## SPECIALIST MATHEMATICS Year 12

Section Two:

### Calculator-assumed

Your name \_\_\_\_\_

Teacher's name \_\_\_\_\_

### Time and marks available for this section

Working time for this section: 30 minutes

Marks available: 39 marks

### Materials required/recommended for this section

#### ***To be provided by the supervisor***

This Question/Answer Booklet

Formula Sheet

#### ***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

**Question 1**

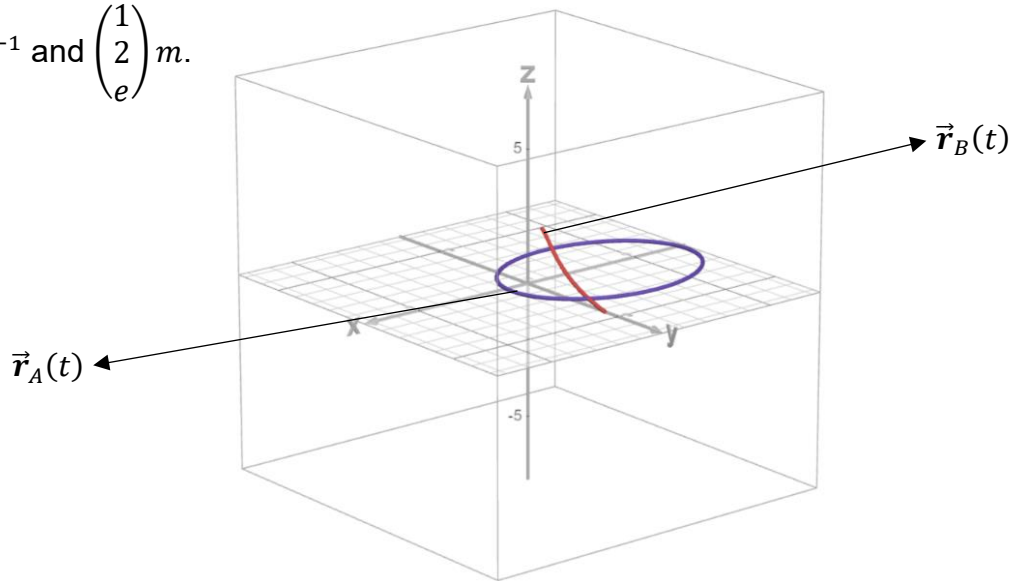
**(5 + 5 + 4 = 14 marks)**

Two particles A and B, are travelling within the same Cartesian plane, as shown below. The acceleration vectors for both particles are  $\vec{a}_A(t) = \begin{pmatrix} -4 \cos(t) \\ -3 \sin(t) \\ 0 \end{pmatrix}$  and

$$\vec{a}_B(t) = \begin{pmatrix} e^{\sin(t)}(1 - \sin^2(t) - \sin(t)) \\ 2e^t \\ -\cos(t)e^{\cos(t)} + (-\sin^2(t)e^{\cos(t)}) \end{pmatrix},$$

where A initially had a velocity and displacement of  $\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \text{ms}^{-1}$  and  $\begin{pmatrix} 4 \\ \frac{3}{2} \\ 2 \end{pmatrix} \text{m}$  and B initially had a velocity and displacement of  $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \text{ms}^{-1}$  and  $\begin{pmatrix} 1 \\ 2 \\ e \end{pmatrix} \text{m}$ .

displacement of  $\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \text{ms}^{-1}$  and  $\begin{pmatrix} 4 \\ \frac{3}{2} \\ 2 \end{pmatrix} \text{m}$  and B initially had a velocity and displacement of  $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \text{ms}^{-1}$  and  $\begin{pmatrix} 1 \\ 2 \\ e \end{pmatrix} \text{m}$ .



a) Hence, show that the displacement function of particle A, is represented as

$$\vec{r}_B(t) = \begin{pmatrix} e^{\sin t} \\ 2e^t \\ e^{\cos t} \end{pmatrix}$$

b) Determine if the particles A and B will collide in  $2\pi$  seconds, and if so, where, and when?

c) Prove that particle A is not in circular motion.

**Question 2****(5 + 1 = 6 marks)**

Three objects have the given equations represented and graphed below.

$$2x^2 + y^2 + z^2 = 5$$

$$x^2 + y^2 + z^2 = 3$$

$$x^2 + 2y^2 - z^2 = 4$$

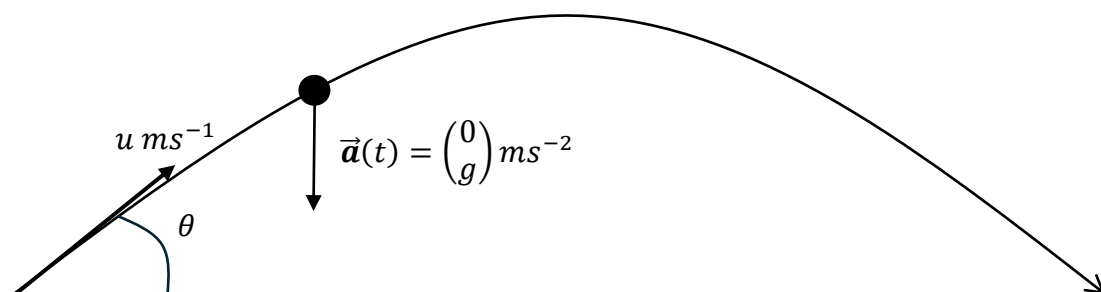
- a) Hence, by showing all necessary steps, find the coordinates of all intersections of the three objects.

- b) Hence, find the cartesian equation of the plane, containing all those points.

## Question 3

(5 + 3 + 4 = 12 marks)

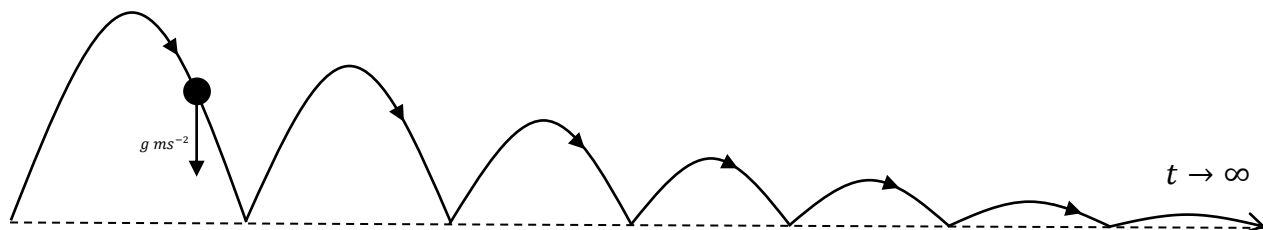
A man threw a tennis ball, with a speed of  $u \text{ ms}^{-1}$  at an angle of  $\theta^\circ$  above the horizontal. The force acted on the ball is its gravitational force, which will have a constant acceleration of  $g \text{ ms}^{-2}$ .



a) Show that the maximum height  $h_{max}$  is expressed as follows.

$$h_{max} = \frac{-u^2 \sin^2 \theta}{2g}$$

As the tennis ball hits the ground once, it continuously bounces until the ball is fully at rest. This is depicted at the diagram below.



As  $t \rightarrow \infty$ , the maximum height of the tennis ball is going to decrease gradually. It is estimated that for each bounce, it vertically rebounds to 75% of the height from which it fell.

- b) Hence, show that the total height travelled of the tennis ball before coming to rest,  $d_{total}$  is the expression shown below.

$$d_{total} = -\frac{4u^2 \sin^2 \theta}{g}$$

- c) Show that the velocity increases for each bounce, assuming the angle  $\theta$  is constant for each bounce.

**Question 4**

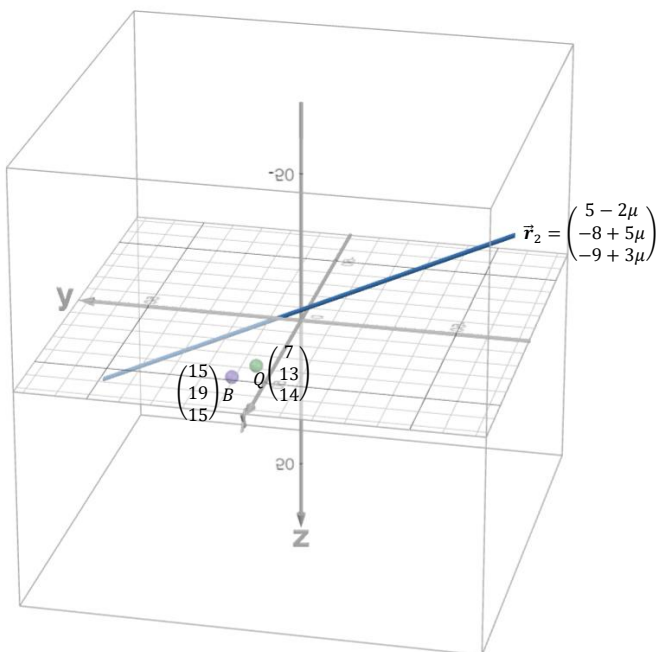
**(7 marks)**

The position vectors are follows as  $\overrightarrow{OQ} = \begin{pmatrix} 7 \\ 13 \\ 14 \end{pmatrix}$  and  $\overrightarrow{OB} = \begin{pmatrix} 15 \\ 19 \\ 15 \end{pmatrix}$ . The line  $\vec{r}_1$  passes through these coordinates, and the line  $\vec{r}_2$  has the vector equation.

$$\vec{r}_2 = \begin{pmatrix} 5 - 2\mu \\ -8 + 5\mu \\ -9 + 3\mu \end{pmatrix}$$

All the information provided is presented in a Cartesian plane.

By proving that  $\vec{r}_2$  and  $\vec{r}_1$  are non-intersecting, find the shortest distance between both lines without using cross-product and only using the scalar projection method (*Anyone who does otherwise will get one mark*).



**END OF CALCULATOR-ASSUMED**